Exercise 21

Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram like Figure 5 showing the line segments with lengths dx, dy, and Δy.

$$y = \sqrt{x-2}, \quad x = 3, \quad \Delta x = 0.8$$

Solution

Compute the derivative of y.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\sqrt{x-2} \\
&= \frac{d}{dx}(x-2)^{1/2} \\
&= \frac{1}{2}(x-2)^{-1/2} \cdot \frac{d}{dx}(x-2) \\
&= \frac{1}{2}(x-2)^{-1/2} \cdot (1) \\
&= \frac{1}{2\sqrt{x-2}}
\end{aligned}$$

Consequently, the differential of $y = \sqrt{x-2}$ is

$$dy = \frac{1}{2\sqrt{x-2}}\,dx,$$

so when $x = 3$ and $\Delta x = dx = 0.8$,

$$dy = \frac{1}{2\sqrt{3-2}}(0.8) = 0.4$$

$$\Delta y = y(3+0.8) - y(3) = \sqrt{(3+0.8)-2} - \sqrt{3-2} \approx 0.341641.$$

The function is plotted below along with its tangent line at $x = 3$.